BELLCOMM, INC.
955 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D. C. 20024

B70 10046

SUBJECT: Selection in a Limited Resource Environment
Case 105-4

DATE: October 22, 1970

FROM: E. D. Marion

ABSTRACT

Long range planning for earth orbital experiment programs often involve use of the idea that experiments, on the average, display a certain cost per pound of experiment weight in orbit. This memo presents a different approach to the problem of estimating experiment weights and costs, based on a statistical analysis of experiment selection. It reflects primarily the psychology of the selection process and is favorably correlated with several examples of experiment selection. An example is presented to show how future shuttle traffic for OSSA payloads could be estimated.

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MEMORANDUM FOR FILE

Introduction

A manager in charge of a large research laboratory is annually faced with the problem of distributing his limited research budget among the almost unlimited number of projects clamoring for money. If you examine the final results of his decisions you will probably find a large number of inexpensive project items, several of intermediate cost and a few that are quite expensive.

The process illustrated is one of choice or selection in an environment where resources are limiting. And the results are only common sense - the cheaper something is, the more likely it is to be bought.

If this tendency can be quantized, then it might be applicable say to the distribution of experiments, by cost, in a research laboratory, or to the distribution by weight of experiments that are flown on a payload limited rocket.

This memo hypothesizes such a quantitative relationship and applies it to some aerospace situations. The results, although certainly not conclusive, definitely tend to verify the hypothesis.

Hyperbolic Selection Rule

Let us hypothesize that the probability that an item (e.g. an experiment) will be selected is inversely proportional to the cost. So;

$$P(c) \propto \frac{1}{c} \tag{1}$$

where P (c) is the probability that some item, any item, of cost c, will be selected. If all the items selected are put in a bag, then P (c) may also be viewed as the probability of finding an item of cost c, if we randomly draw one item from the bag. Hence the cost distribution in the resultant collection of items will also follow the hyperbolic assumption. Thus:

$$n(c) = \frac{K}{c} \tag{2}$$

The relationship between equations (1) and (2) is explored more fully in Appendix I.

where K is a proportionality constant, and n (c) is the frequency function (Reference 1) for the selected items.

In a given range of costs, say δc, the number of items to be expected is:

$$\delta N(c) = n(c)\,\delta c = \frac{K\delta c}{c} \tag{3}$$

since c is the unit cost of those $\delta N(c)$ items, the total cost of the items is:

$$\delta C = \delta N(c)\ c = K\delta c \tag{4}$$

and the total cost of all the items in the range of costs between c_1 and c_2 is

$$C_t = \int_{c_1}^{c_2} \delta N(c)\,c = \int_{c_1}^{c_2} K\delta c = K(c_2 - c_1) \tag{5}$$

Since this process is dealing with limited resources, and people seldom, if ever, <u>under</u> spend, then C_t must be the total amount of resources available for the endeavor. It is the budget, B. It follows then that the proportionality constant, K, can be defined in terms of the budget by using equation 5.

$$K = \frac{C_t}{c_2 - c_1} = \frac{B}{c_2 - c_1} \tag{6}$$

Substituting this into equation 2 gives

$$n(c) = \frac{B}{c(c_2 - c_1)} \tag{7}$$

The cumulative number of items between the minimum cost, c_1, and any intermediate cost c can be computed by integrating N(c). Thus from equation 3.

$$\Sigma_N = \int_{c_1}^{c} n(c)\quad \delta c = \int_{c_1}^{c} \frac{B\delta c}{c(c_2 - c_1)} \tag{8}$$

Taking the term $\frac{B}{c_2-c_1}$ outside the integral, since it is a constant (equation 6), gives

$$\Sigma_N = \frac{B}{c_2-c_1} \int_{c_1}^{c} \frac{\delta c}{c} = \frac{B}{c_2-c_1} \ln \frac{c}{c_1} \tag{9}$$

where c may take any value between c_1 and c_2. If $c = c_2$ then Σ_N becomes the total number of items in the collection, N_T.

Data Comparisons

To test the validity of these equations, some samples of selection in a restricted resource environment need to be examined. The experiments for Skylab represent such a sample. They were selected from a large list of candidates that were all competing for the limited funds available for experiments. According to equation 9, a plot of Σ_N vs ln c should be a straight line. This data from References 2 and 3 is currently out of date but represents a snapshot of the selection process of one point in time. It is shown on figure 1. The data does indeed correlate well with a straight line.

The OSSA prospectus, reference 4, provides another opportunity to check the correlation. In this case the items are payloads, and the cost data is accumulated over the past several years.

Remember that the distribution function relates to the choice or selection from competing candidates. In funding payloads, the choice initially is related to the cost of development rather than the cost of building the payload. Because of the development costs, the percent difference in total program cost between building the first payload and building the first two payloads can be quite small. In the same program, when later payloads are added, the decision is based simply on the cost of the additional payloads and the development costs need not be considered. This kind of visibility into the funding history was not always available, so the data was treated as follows:

1) Experiments where the costs were shared by other agencies were not considered. These are in limited supply and are not OSSA resource limited.

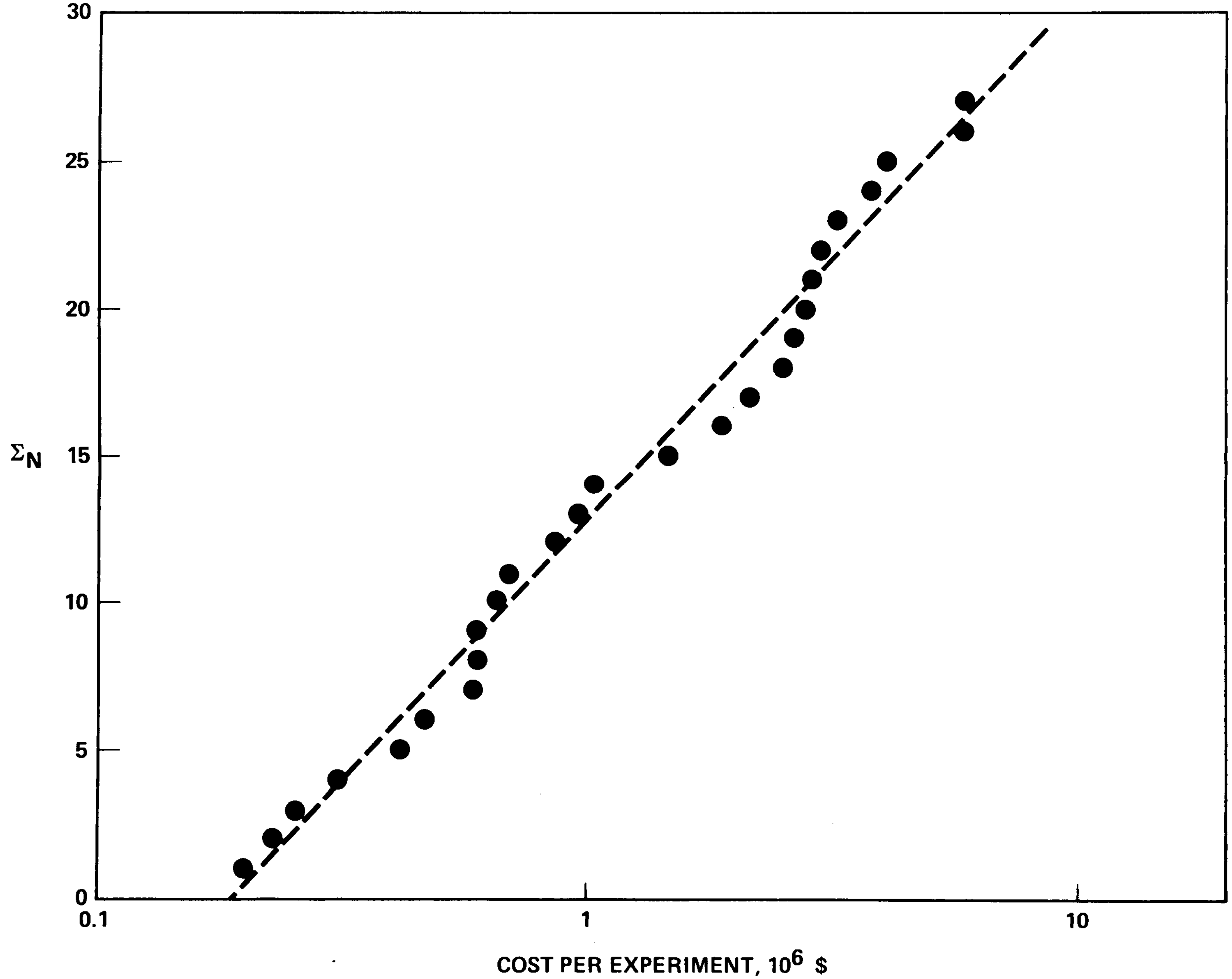


FIGURE 1 - SKYLAB EXPERIMENT COSTS

2) Launch vehicle costs are taken from a different account and are not considered.

3) All data was normalized on a cost per payload basis. If the total number of payloads built in a particular program was N, the total program costs are divided by N to determine cost per payload.

4) Where the decision to buy additional payloads was identified, the added cost for the additional payloads was used in computing cost per payload.

Because the first decision to begin a particular payload program was not always described, some scatter in the data will be introduced by the process of computing cost per payload. The data is shown in figure 2. The Viking point is almost a self explaining anomaly.

Another resource that is commonly limited in space program applications, is weight in orbit. This implies that a plot of Σ_N versus ln (weight) for space experiments should be a straight line. This cannot be done for OSSA payloads because they tend to be grouped according to the payload capacity of the launch vehicles. However, the Skylab internal experiments do represent an appropriate data set, and these are shown on figure 3. For comparison, the correlation between experiment cost and experiment weight is shown on figure 4 for the Skylab experiments. Clearly figures 1 and 3 are <u>not</u> interrelated by a \$/LB relationship.

If the data on figures 1, 2, and 3 are really described by equation 9, then the slopes must be described by $\frac{B}{c_2-c_1}$ where

- B is set equal to the total amount of resource expended on the experiments or payloads.
- c_2 is set equal to the maximum cost or weight c_{max} of the items listed.
- c_1 is set equal to the minimum cost or weight c_{min} of the items listed.

The values for B, c_{max}, and c_{min}, are listed in Table I for the Skylab experiment weights and costs, and for the OSSA costs. Raw data for these two sources is given in Appendix 2.

LAUNCH AND RECOVERY WINDOWS; 2 HRS BEFORE SUNRISE
UNTIL 2 HRS BEFORE SUNSET

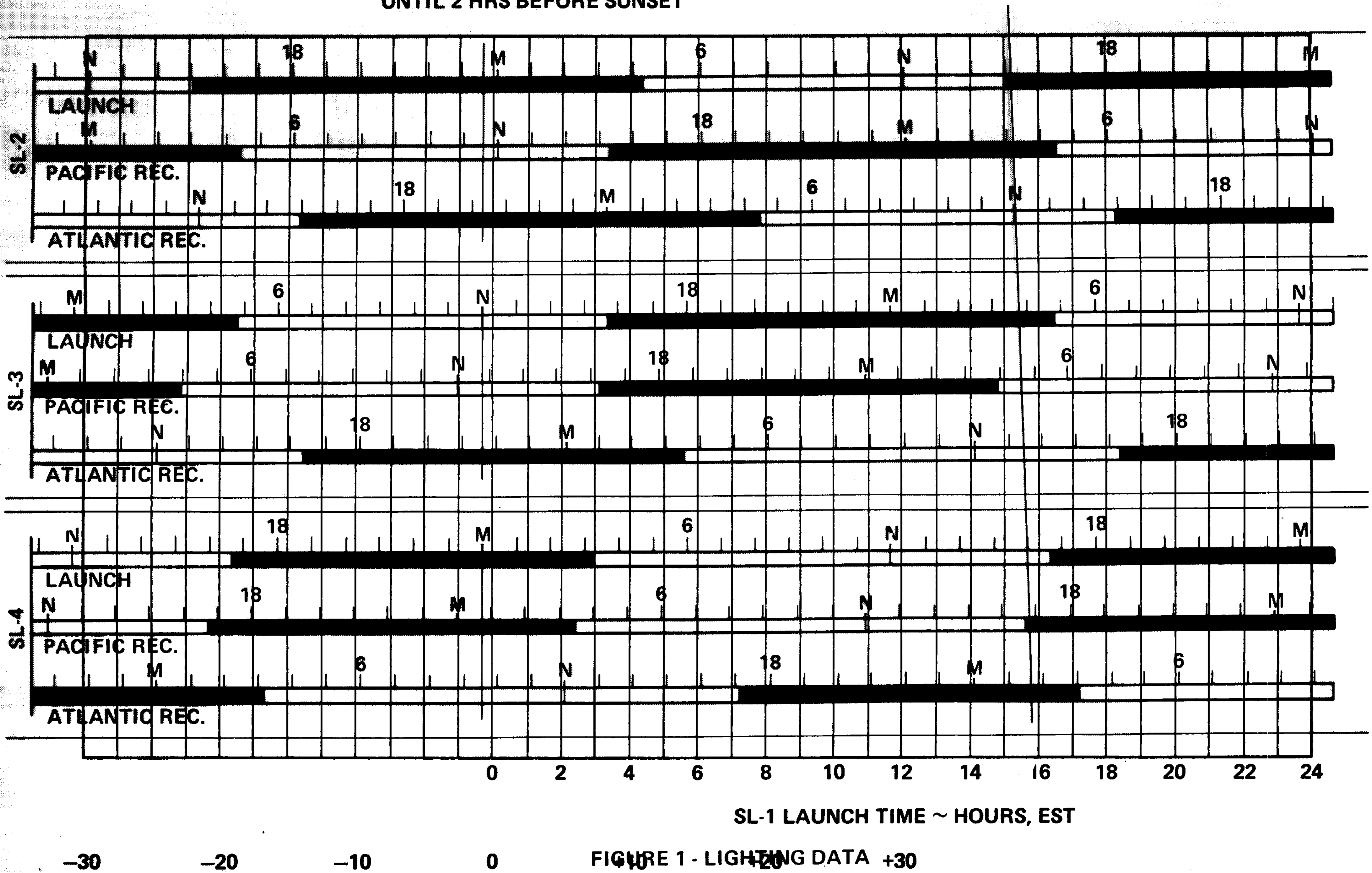


FIGURE 1 - LIGHTING DATA

FIGURE 3 - CURSOR

FIGURE 2 - BASE

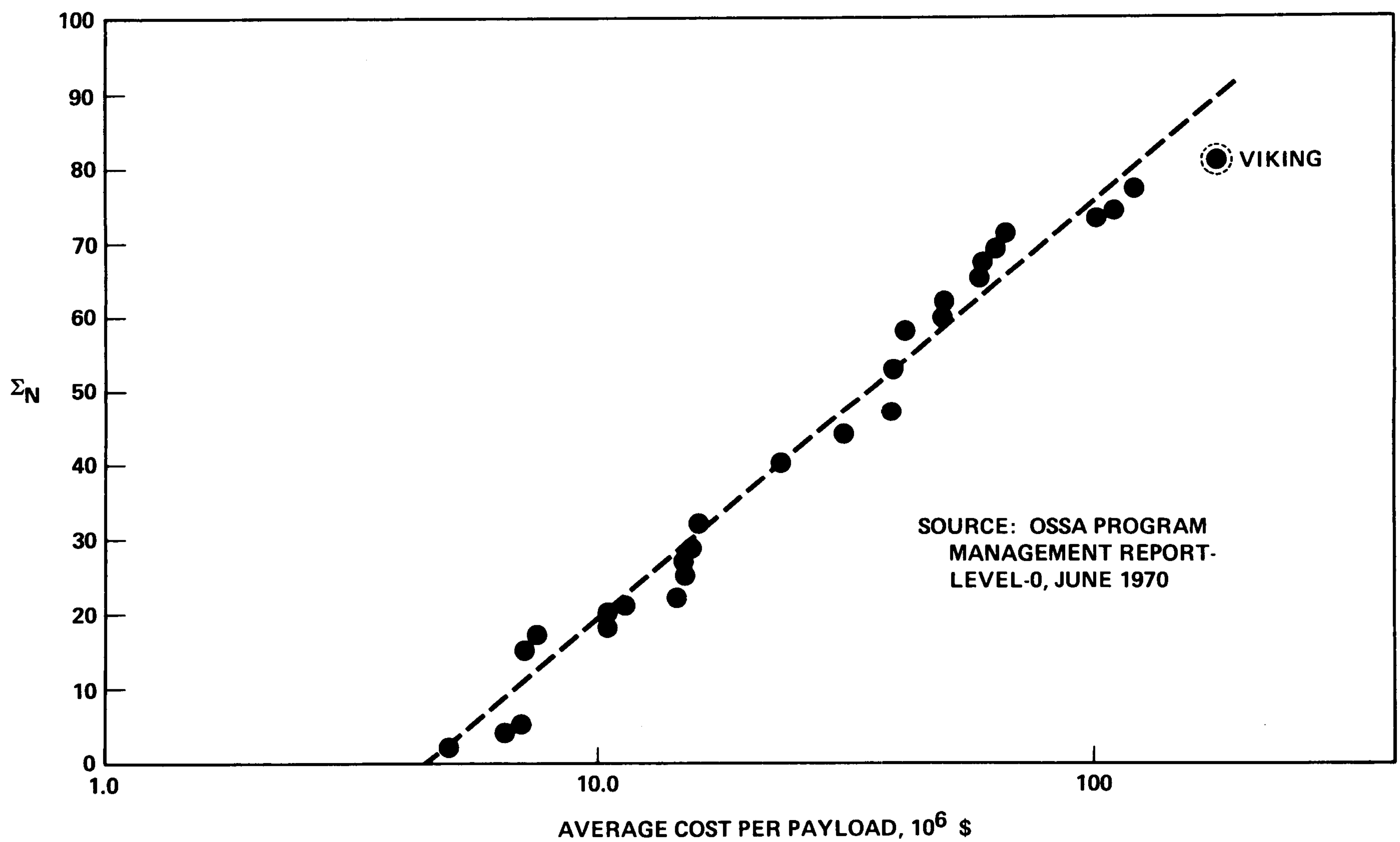


FIGURE 2 - OSSA DOMESTIC PAYLOADS

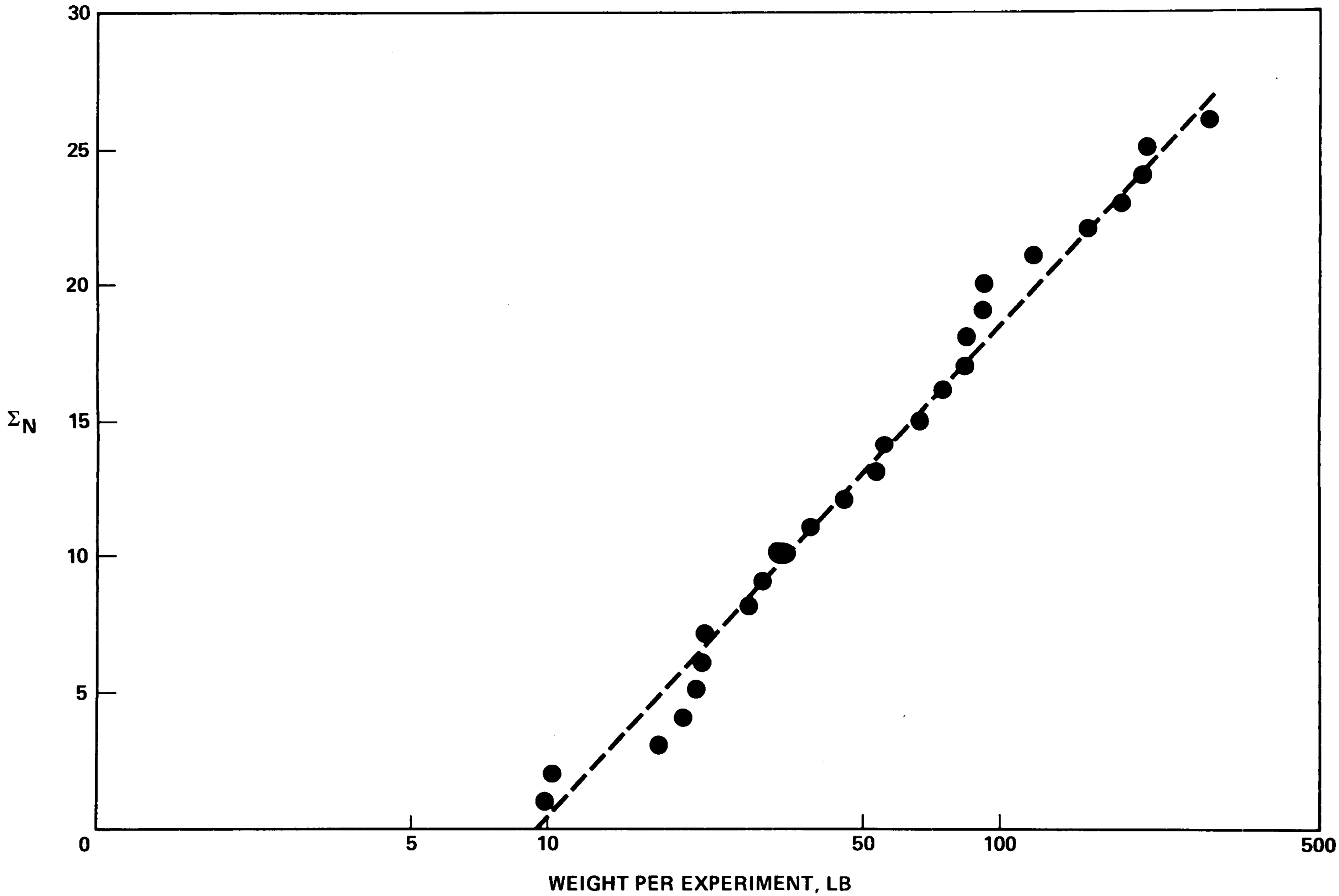


FIGURE 3 - SKYLAB EXPERIMENT WEIGHTS

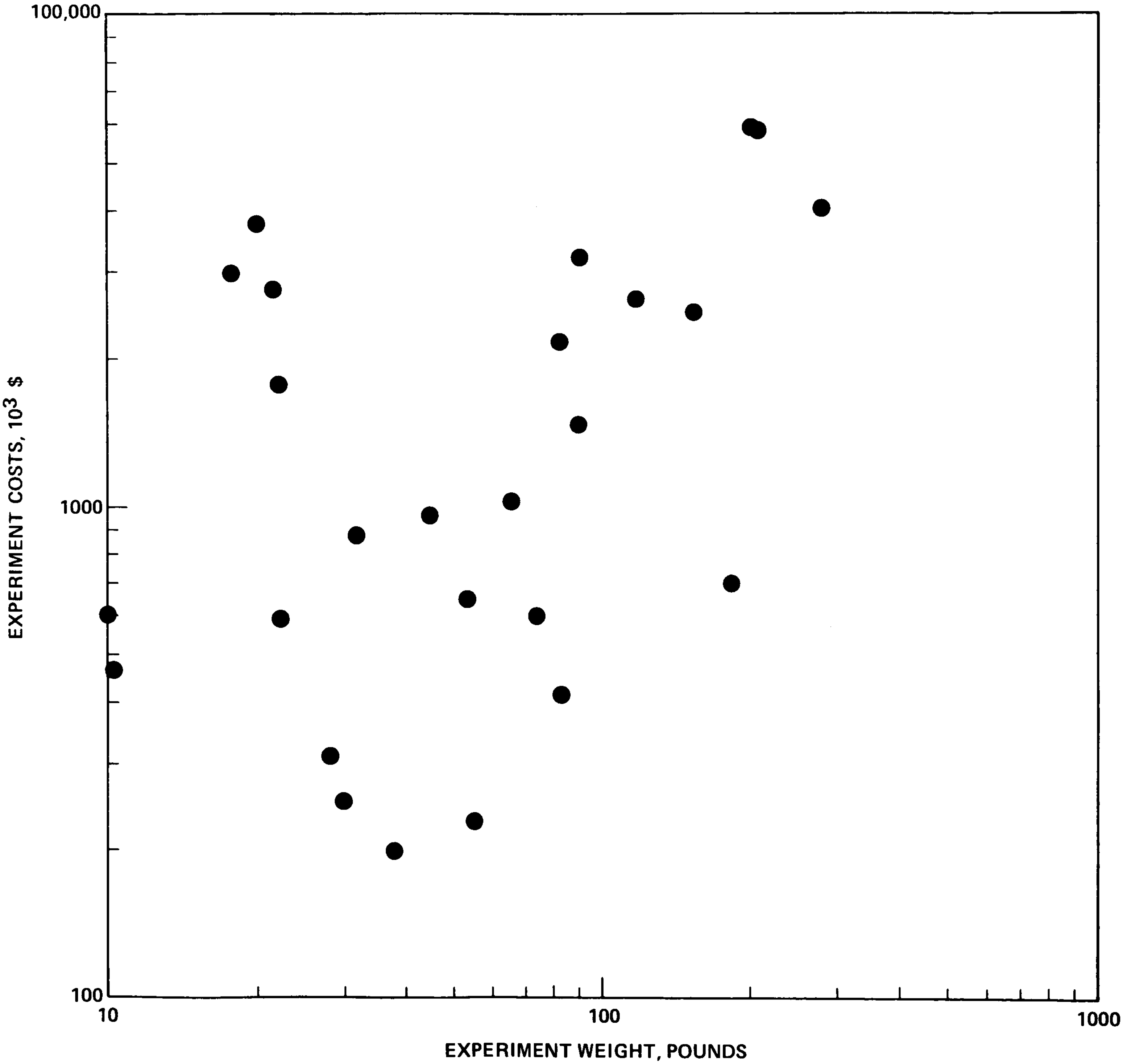


FIGURE 4 - SKYLAB EXPERIMENTS \$/POUND

The validity of $\frac{B}{c_2-c_1}$ as a slope can be checked by using equation 9 to compute the total sample size, N_T, and comparing this with actual values.

If the actual minimum and maximum costs per experiment (c_{min} and c_{max}) in the example are used, then Σ_N from equation 9 will be the number of items between c_{max} and c_{min}, and so will not include the actual items corresponding to c_{max} and c_{min}. This discrepancy is a result of the fact that the distribution has been treated mathematically as a continuum, but is actually a series of discrete increments. The deviation is only significant for values of Σ_N near 1 which corresponds to values of c near c_1.

There are 3 ways to adjust for this deviation. One way to compensate for this is to use values of c_1 that are extended slightly, say 10%, beyond the actual limits of the data. A second possibility is to simply add 1 to the computed Σ_N as a correction for the region of small C's.

A third more rigorous approach is to define the value of c_1 such that when $C = c_{min}$ is inserted into equation 9 the resultant Σ_N is $\underline{1}$. Thus

$$\Sigma_N = 1 = \frac{B}{c_{max}-c_{min}} \ln \frac{c_{min}}{c_1}$$

in most cases $c_{min} \ll c_{max}$ and the difference between the two can be simply treated as c_{max}. Thus

$$\frac{c_{max}}{B} = \ln \frac{c_{min}}{c_1} \text{ or } \exp\left(\frac{c_{max}}{B}\right) = \frac{c_{min}}{c_1}$$

and $$c_1 = c_{min} \exp\left(\frac{-c_{max}}{B}\right)$$

These three approaches are compared in Table I, where the actual N_T and the computed N_T's are compared. Agreement is generally good.

<u>Applications</u>

The analysis can provide some useful guidance in estimating future traffic models for space experiments. Let us assume that OSSA's future budget during the time of the

TABLE I

SAMPLE	c_{min}	c_{max}	B	Actual N_T	$c_1=0.9c_{min}$	COMPUTED $\Sigma_n=\Sigma_n+1$	NT $c_1=c_{min}\ e^{-\left(\frac{c_{max}}{B}\right)}$
Skylab I Experiment Weights, LBS	10	283	2037	26	26	26	26
Skylab I Experiment Costs, $\$10^3$	200	5916	47154	26	29	29	29
OSSA Payload Costs, $\$10^6$, without viking	5	121	2633	77	74	73	73
OSSA Payload Costs $\$10^6$, with Viking	5	178	3344	81	71	70	70

TABLE I-A

SKYLAB INTERNAL EXPERIMENT SUMMARY

	WEIGHT, LBS.		COST, 10^3\$	
EXPERIMENT	Σ_N	C	Σ_N	C
A	1	10.0	8	600
B	2	10.2	6	467
C	3	18.0	21	3000
D	4	20.0	23	3784
E	5	21.5	20	2794
F	6	22.0	16	1702
G	7	22.1	7	594
H	8	28.0	4	310
I	9	30.0	3	252
J	10	32.0	12	870
K	11	38.0	1	200
L	12	45.0	13	962
M	13	53.0	10	655
N	14	55.0	2	230
O	15	66.0	14	1028
P	16	74.0	9	600
Q	17	82.0	17	2160
R	18	83.0	5	414
S	19	90.0	15	1476
T	20	90.5	22	3248
U	21	118.0	19	2688
V	22	153.0	18	2500
W	23	184.0	11	700
X	24	201.0	26	5916
Y	25	208.0	25	5840
Z	26	283.0	24	4074

Experiment names have been deleted in order to desensitize the data.

space shuttle is about 600 million dollars, of which about 2/3 will be spent on earth orbit experiments. So

$$B_{\$} = 400 \times 10^6 \ \$/yr$$

Based on reference 4, the cost per payload will range between 4×10^6 to 150×10^6 dollars, so

$$c_2 = 150 \times 10^6 \ \$, \ c_1 = 4 \times 10^6 \$$$

Equation 9 then says that approximately 10 payloads per year will be purchased.

Assuming that the shuttle payload capacity is about 25,000 lb, then, with 5000 lbs allotted for the experimentors, about 20,000 lbs should be available for experiments. Hence

$$B_{wt} = 20{,}000 \text{ lb/shuttle flight}$$

Since in some instances this full capacity may be allotted to one experiment, c_2 = 20,000 lb. Judging by Skylab and OSSA data, a reasonable estimate for minimum experiment weight is about 100 lbs, so c_1 = 100 lb. Equation 9 then says that, on the average, about 5 experiments will be carried per shuttle flight. Combining these two results indicates that OSSA will use about 2 shuttle flights per year.

Remember that this is just an example of the analytical approach, and the input values are simple assumptions. The inputs require more careful selection before conclusions are drawn from the results.

Conclusions

The hyperbolic selection rule is really a description of the psychology of the selection process. As such it should be applicable to any situation where the proper conditions are met. At this level of analysis these conditions are little more than opinion, however the following requirements seem appropriate.

1) The selectors must be aware of the resource restriction.

2) The supply of candidate items must be so big that some are left unchosen at all cost levels.

3) The final list of selected items must contain several items, certainly more than one or two

The data shown here meets these criteria. And although it does represent a limited set of test cases, it does clearly support a hyperbolic distribution function for limited resource selection processes.

E D Marion

1013-pak

E. D. Marion

REFERENCES

1. Introduction to Mathematical Statistics, Paul G. Hoel, Wilen & Sons, Inc. 1954.

2. AAP Weight & Performance Report, NASA, January 1970.

3. AAP Presentation to 69-6 MSFEB:AAP Experiment Development Cost Review, December 1, 1969.

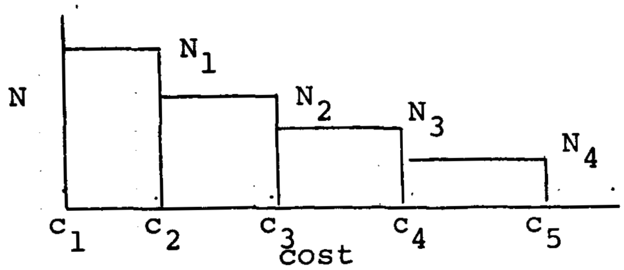
4. Space Science and Applications - Program Management Report, Level 0, June 1970.

APPENDIX I - DISTRIBUTION

FUNCTIONS

APPENDIX I - DISTIRBUTION FUNCTION

Statistical distributions are usually presented in the form of a histogram, such as:



Here there are N_1 items that cost more than c_1 and less than c_2. If the sample width, c_2-c_1, were smaller, then the number of items in each sample would be correspondingly smaller. A first approximation that becomes more accurate as the sample width gets smaller, is to assume that, at any given cost level, the number of items in the sample is directly proportional to the sample width. That is

$$N_i = K_1 (c_{i+1} - c_i) \text{ or } K_1 = \frac{N_i}{c_{i+1}-c_i} \qquad \text{(A-1)}$$

The constant K_1 in this equation, when the sample width is differentially small, is called the sample density, n, and is the number of samples per unit sample width.

In the correlation suggested on the first page of this memo the distribution function is

$$n(c) = \frac{K}{c} \qquad \text{(A-2)}$$

In a collection of items, with costs ranging from c_o to c_F, the total number of items is:

$$N_T = \int_{c_o}^{c_F} n(c)dc = \int_{c_o}^{c_F} \frac{K}{c} dc = K\ln\left(\frac{c_F}{c_o}\right) \qquad \text{(A-3)}$$

What is the probability of finding an item in this collection of N_T items, whose cost lie in the range between c and c+Δc.

The total number of items between the costs of c and c+Δc is:

$$N_c = \int_c^{c+\Delta c} n(c)dc = K\ln\left(\frac{c+\Delta c}{c}\right) \qquad \text{(A-4)}$$

Thus the probability of finding an item in the desired range is:

$$P(c) = \frac{N_c}{N_T} = \frac{K\ln\left(\frac{c+\Delta c}{c}\right)}{K\ln\frac{c_F}{c_o}} \qquad \text{(A-5)}$$

Ignoring the constants:

$$P(c) \;\alpha\; \ln\left(\frac{c+\Delta c}{c}\right) = \ln\left(1+\frac{\Delta c}{c}\right) \qquad \text{(A-6)}$$

As Δc becomes small compared to c, and $\frac{\Delta c}{c} << \underline{1}$

the log term can be approximated by:

$$\ln\left(1+\frac{\Delta c}{c}\right) \simeq \frac{\Delta c}{c} \qquad \text{(A-7)}$$

and hence

$$P(c) \;\alpha\; \frac{\Delta c}{c} \quad \text{or} \quad P(c) \;\alpha\; \frac{1}{c}$$

thus the statements

$$P(c) \;\alpha\; \frac{1}{c}$$

and $n(c) = \frac{K}{c}$, on page 1

are mathematically consistant.

APPENDIX 2 - COST DATA

TABLE II-A

OSSA DATA SUMMARY

NAME	WT	COST	# OF EXPERIMENTS	C $/EXP	Σ_N
Helios	500	10.0	2	5.0	2
SSS	114	13.0	2	6.5	4
EASEP		7.0	1	7.0	5
Explorers	600	71.0	10	7.1	15
ISIS	575	15.0	2	7.5	17
SAS-B	340	10.4	1	10.4	18
RAE-B	725	21.0	2	10.5	20
SAS-A	320	11.3	1	11.3	21
SAS-C	340	14.3	1	14.3	22
INTELLSAT I, II, III	380	45.0	3	15.0	25
SMS	500	30.0	2	15.0	27
GEOS	465	35.0	2	15.5	29
ATMOS EXP	1000	48.0	3	16.0	32
OSO	1000	187	8	23.4	40
ALSEP A, B, C, AZ	284	125	4	31.2	44
Nimbus	1465	117	3	39.0	47
OGO		236	6	39.3	53
Nimbus B	1465	210	5	42.0	58
ALSEP D, E	284	100	2	50.0	60
Pioneer F&G	550	100	2	50.0	62
ATM	21,331	177	3	59.0	65
Mariner Mars 71	2200	120	2	60.0	67
ERTS	1400	127	2	63.5	69
ATS D&E	1749	133	2	66.5	71
ATS F&G	2050	201	2	100.5	73
Mariner Venus/Mer	925	110	1	11.0	74
OAO	4660	363	3	12.1	77
Viking*	2400*	711	4*	177.7	81
		3344**			

*2 Spacecraft ea. vehicle, 1 orbiter, 1 lander
2 vehicles in program

** Total Cost without Viking = 2633